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Short Papers

Even- and Odd-Mode Impedances of Coupled Elliptic Arc Strips

B. N. DAS AND K. V. S. V. R. PRASAD

Abstract —**A derivation of the expression for even- and odd-mode impedances of coupled elliptic arc strips between grounded, confocal elliptic cylinders, and above a grounded elliptic cylinder, symmetrically located with the minor axis, is presented. The analysis is based on TEM-mode approximation. The Green's function formulation is used to obtain variational expressions for the even- and odd-mode capacitances for the more general case of different dielectrics on the two sides of the coupled strips. Numerical results are presented for coupled elliptic and circular cylindrical arc strips. It is also shown that the formulation can be used to find the effect of environmental changes on an otherwise planar structure.**

I. INTRODUCTION

Some investigations on elliptic and circular cylindrical striplines have been reported in the literature [1]–[4]. The impedance of warped lines can be determined from the results of the analysis of such lines by assuming that the radii of curvature are very large and the arc lengths remain finite. Wang [1] presented impedance data of cylindrical and cylindrically warped strip- and microstrip lines from numerical solution of Laplace's equation. He used the results of the analysis to calculate the effect of environmental changes on the impedance of an otherwise planar structure. The numerical results presented by him show a marked deviation from those calculated using the formula for planar structure. From physical considerations, however, it may be concluded that the impedance of a warped line should not differ appreciably from that of an otherwise planar structure. It has been established that the results obtained by Wang for warped lines are not correct [2], [4]. It is worthwhile to investigate the effect of environmental changes on the even- and odd-mode impedances of an otherwise planar structure. Expressions for these impedances can be obtained from the analysis of two coupled elliptic arc strips between confocal elliptic grounded cylinders. To the best of the author's knowledge, no investigation on coupled arc strips between grounded elliptic and circular cylinders or above such surfaces has been reported in the literature.

In the present work, a method of derivation of the expressions for the even- and odd-mode impedances of coupled elliptic and circular cylindrical arc strips between two dielectric layers is presented. If the transverse dimensions of the structure are small compared to the operating wavelength, quasi-TEM-mode approximation can be used for the analysis. The potential function for the even- and odd-mode configurations is derived using a Green's function formulation and TEM-mode approximation. Variational expressions for the even- and odd-mode capacitances are found, assuming suitable charge distribution on the arc strips. Formulation is made for the general case of elliptic arc strips between two confocal grounded elliptic cylinders. The corresponding expressions for the case of coupled elliptic arc strips above a grounded elliptic cylinder are found by assuming that the upper cylinder is moved to infinity. The even- and odd-mode impedances of coupled cylindrical strip- and microstripline are then found from the analysis of the elliptic line by assuming that eccentricity is equal to zero.

Numerical results on the even- and odd-mode impedances for 1) coupled elliptic arc strips between grounded elliptic cylinders

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and above a grounded elliptic cylinder, and 2) circular arc strips between grounded circular cylinders and above a grounded cylinder are presented. The effect of environmental changes on an otherwise planar structure is found by the method suggested in the literature [4]. A comparison between the numerical results on such warped lines and planar structure is presented.

II. Derivation of the Expressions for Even- and Odd-Mode Impedances

Fig. 1 shows two elliptic arc strips AB and CD located at the interface between two dielectric media having relative dielectric constants ϵ_1 and ϵ_2 sandwiched between two earthed elliptic cylinders P and Q. The interface between the two dielectrics is an ellipse and all the three ellipses of Fig. 1 are confocal. It is assumed that there are electric walls at $\eta = \eta_1$ and $\eta = (\pi - \eta_1)$. For a TEM-mode approximation, the Poisson equation relating the potential function V to the charge density ρ is given by [4]

$$\frac{\sqrt{(1-\Psi^2)(\Phi^2-1)}}{k^2(\Phi^2-\Psi^2)}\left[\frac{\partial}{\partial\Phi}\left(\sqrt{\frac{\Phi^2-1}{1-\Psi^2}}\,\frac{\partial V}{\partial\Phi}\right)+\frac{\partial}{\partial\Psi}\left(\sqrt{\frac{1-\Psi^2}{\Phi^2-1}}\,\frac{\partial V}{\partial\Psi}\right)\right]=-\frac{\rho}{\epsilon}(\Phi,\Psi) \quad (1)$$

where Φ = constant and Ψ = constant represent, respectively, the sets of confocal ellipses and orthogonal hyperbolas. ϵ is the dielectric constant of the material and k is the focal distance. The solution of (1) is found for a source in the form of a line charge per unit length located at (Φ_o, Ψ_o). Assuming that the charge is distributed over a surface whose cross section is in the form of an elementary curvilinear square of dimension $\Delta\Phi = \Delta_1$ and $\Delta\Psi = \Delta_2$, centered at (Φ_o, Ψ_o), the charge density $\rho(\Phi, \Psi)$ is given by

$$\rho(\Phi,\Psi) = \tau(\Psi)/h_1 h_2 \Delta_1 \Delta_2$$

where

$$h_1 = k\sqrt{\frac{\Phi^2-\Psi^2}{\Phi^2-1}}$$

$$h_2 = k\sqrt{\frac{\Phi^2-\Psi^2}{1-\Psi^2}}.$$

With the substitution

$$\Phi = \operatorname{Cosh}\xi \quad (2a)$$

$$\Psi = \operatorname{Cos}\eta \quad (2b)$$

then (1) reduces to

$$\frac{\partial^2 V}{\partial \xi^2} + \frac{\partial^2 V}{\partial \eta^2} = -\tau/\epsilon\Delta\xi\Delta\eta. \quad (2c)$$

The elliptic boundaries shown in Fig. 1 are given by

$$\xi_1 = \operatorname{Cosh}^{-1}(a_1/k) \quad (3a)$$

$$\xi_o = \operatorname{Cosh}^{-1}\left(\frac{a_1+p_1}{k}\right) \quad (3b)$$

$$\xi_2 = \operatorname{Cosh}^{-1}\left(\frac{a_1+p_2}{k}\right) \quad (3c)$$

where k is the focal distance.

The even- and odd-mode characteristic impedances can be determined from the expression of the form

$$Z = Z_o(C_o/C)^{1/2} \quad (4)$$

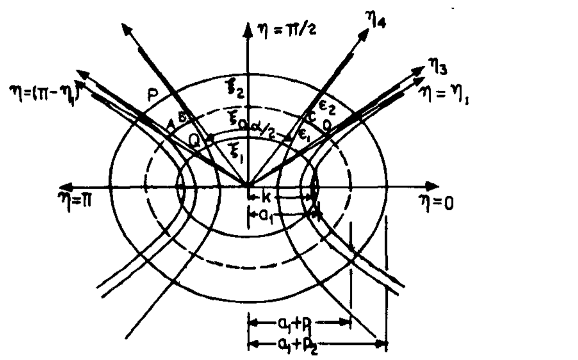

Fig. 1. Coupled elliptic arc strips between two confocal elliptic cylinders

where C and C_o are the line capacitances of the transmission line with and without the dielectrics, respectively, and Z_o is the characteristic impedance of the corresponding air line structure. For an infinitesimally thin conductor located at $\xi = \xi_o$, the variational expression for the line capacitance is given by [5]

$$\frac{1}{C} = \frac{\int\int \rho(\eta')\rho(\eta)G(\xi,\eta|\xi_o,\eta')\,d\eta\,d\eta'}{\left[\int\rho(\eta)\,d\eta\right]^2} \quad (5)$$

where $G(\xi,\eta|\xi_o,\eta')$ is the Green's function.

The charge distribution appearing in (5) can be assumed to be of the form

$$\rho(\eta) = \left[1-\left\{\frac{2}{\Delta}\left[\eta-\left(\eta_3+\frac{\Delta}{2}\right)\right]\right\}^2\right]^{-1/2}, \quad \eta_3 \leqslant \eta \leqslant \eta_4$$

$$= 0, \quad \text{otherwise},$$

where

$$\Delta = \frac{\eta_4-\eta_3}{2}. \quad (6)$$

The expressions for the even- and odd-mode characteristic impedances can, therefore, be determined from the evaluation of the corresponding Green's functions.

For the even-mode configuration, the structure of Fig. 1 gets split into two identical half sections by a magnetic wall of $\eta = \pi/2$. The analysis if carried out by considering the right-half section. The boundary and continuity conditions for the even-mode configuration are given by [6]

$$V(\xi_1,\eta) = 0 \quad (7a)$$

$$V(\xi_2,\eta) = 0 \quad (7b)$$

$$\left.\frac{\partial V}{\partial \eta}\right|_{\eta=\pi/2} = 0 \quad (7c)$$

$$|_{\eta=\eta_1} = 0 \quad (7d)$$

$$\left.\frac{\partial V}{\partial \eta}\right|_{\xi=\xi_o^-} = \left.\frac{\partial V}{\partial \eta}\right|_{\xi=\xi_o^+} \quad (7e)$$

$$\epsilon_0\epsilon_1\left.\frac{\partial V}{\partial \xi}\right|_{\xi=\xi_o^-} = \epsilon_0\epsilon_2\left.\frac{\partial V}{\partial \xi}\right|_{\xi=\xi_o^+}. \quad (7f)$$

For the odd-mode structure, the magnetic wall at $\eta = \pi/2$ is replaced by an electric wall in Fig. 1. The analysis is carried out by considering the right-half section. The boundary and continuity conditions are the same as above, except (7c) is replaced by the equation

$$V|_{\eta=\pi/2} = 0. \quad (7g)$$

Equation (2c) is solved using (7a)–(7f) and assuming that

$\rho(\xi_o, \eta) = \delta(\eta - \eta_o)$ to obtain the even-mode Green's function as

$$G(\xi, \eta|_{\xi_o}, \eta') = -\sum_{n=o}^{\infty} \frac{4}{(2n+1)\pi\epsilon_0\Delta_{n_e}} \operatorname{Sinh}(m_e(\xi_2 - \xi_o)) \cdot \operatorname{Sinh}(m_e(\xi - \xi_1)) \operatorname{Sin}(m_e(\eta - \eta_1)) \cdot \operatorname{Sin}(m_e(\eta' - \eta_1)), \qquad \xi_1 \leqslant \xi \leqslant \xi_o \tag{8a}$$

$$= \sum_{n=o}^{\infty} \frac{4}{(2n+1)\pi\epsilon_o\Delta_{n_e}} \operatorname{Sinh}(m_e(\xi_0 - \xi_1)) \cdot \operatorname{Sinh}(m_e(\xi - \xi_2)) \cdot \operatorname{Sin}(m_e(\eta - \eta_1)) \operatorname{Sin}(m_e(\eta' - \eta_1)), \qquad \xi_0 \leqslant \xi \leqslant \xi_2 \tag{8b}$$

where

$$\Delta_{n_e} = [\epsilon_1 \operatorname{Sinh}(m_e(\xi_2 - \xi_0)) \operatorname{Cosh}(m_e(\xi_0 - \xi_1)) + \epsilon_2 \operatorname{Cosh}(m_e(\xi_2 - \xi_1)) \operatorname{Sinh}(m_e(\xi_0 - \xi_1))]$$

and

$$m_e = \frac{(2n+1)\pi/2}{(\pi/2 - \eta_1)}.$$

Using (4), (5), (6), and (8a), an expression for the even-mode impedance is obtained as

$$Z_{oe} = 480 \cdot \left\{ \sum_{n=0}^{\infty} \frac{J_o^2\left(\frac{m_e\Delta}{2}\right) \operatorname{Sin}^2\left[m_e\left(\eta_3 - \eta_1 + \frac{\Delta}{2}\right)\right]}{(2n+1)[\epsilon_1 \operatorname{Coth}[m_e(\xi_o - \xi_1)] + \epsilon_2 \operatorname{Coth}[m_e(\xi_2 + \xi_o)]]} \cdot \sum_{n=0}^{\infty} \frac{J_o^2\left(\frac{m_e\Delta}{2}\right) \operatorname{Sin}^2\left[m_e\left(\eta_3 - \eta_1 + \frac{\Delta}{2}\right)\right]}{(2n+1)[\operatorname{Coth}[m_e(\xi_o - \xi_1)] + \operatorname{Coth}[m_i(\xi_2 - \xi_o)]]} \right\}^{1/2} \tag{9}$$

where $J_o(X)$ is the Bessel function of the first kind of order zero.

Using (2c), (7a), (7b), and (7d)–(7g), and following a similar procedure, an expression for the odd-mode characteristic impedance is obtained as

$$Z_{oo} = 240 \cdot \left\{ \sum_{n=0}^{\infty} \frac{J_o^2\left(\frac{m_o\Delta}{2}\right) \operatorname{Sin}^2\left[m_o\left(\eta_3 - \eta_1 + \frac{\Delta}{2}\right)\right]}{n[\epsilon_1 \operatorname{Coth}[m_o(\xi_o - \xi_1)] + \epsilon_2 \operatorname{Coth}[m_o(\xi_2 - \xi_o)]]} \cdot \sum_{n=0}^{\infty} \frac{J_o^2\left(\frac{m_o\Delta}{2}\right) \operatorname{Sin}^2\left[m_o\left(\eta_3 - \eta_1 + \frac{\Delta}{2}\right)\right]}{n[\operatorname{Coth}[m_o(\xi_o - \xi_1)] + \operatorname{Coth}[m_o(\xi_2 - \xi_o)]]} \right\}^{1/2} \tag{10}$$

where

$$m_o = \frac{n\pi}{(\pi/2 - \eta_1)}.$$

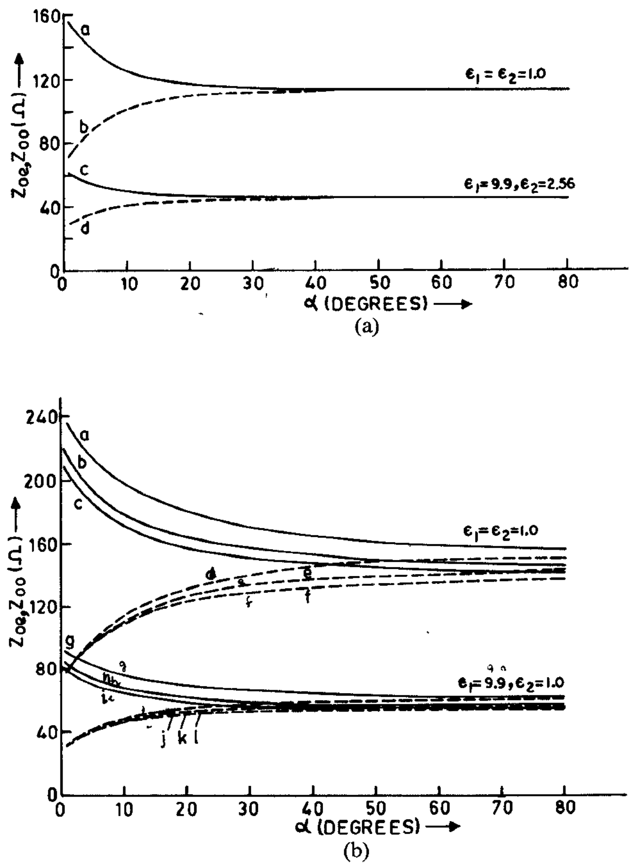

Fig. 2.(a) Variation of even- and odd-mode impedances as a function of α for coupled elliptic striplines, $\eta_4 - \eta_3 = 10^0$. ——— even-mode characteristic impedance, ---- odd-mode characteristic impedance.

Curve	ϵ_1	ϵ_2	P_1/a_1	P_2/a_1	K/a_1	η_1
a	1.0	1.0	0.2	0.5	0.5	0
b	1.0	1.0	0.2	0.5	0.5	0
c	9.9	2.56	0.2	0.5	0.5	0
d	9.9	2.56	0.2	0.5	0.5	0

(b) Variation of even- and odd-mode impedances as a function of α for coupled elliptic microstriplines, $\eta_4 - \eta_3 = 10^0$. ——— even-mode characteristic impedance, ---- odd-mode characteristic impedance.

Curve	ϵ_1	ϵ_2	p_1/a_1	K/a_1	η_1
a, d	1.0	1.0	0.25	0.75	0^0
b, e	1.0	1.0	0.25	0.50	0^0
c, f	1.0	1.0	0.25	0.25	0^0
g, j	9.9	1.0	0.25	0.75	0^0
h, k	9.9	1.0	0.25	0.50	0^0
i, l	9.9	1.0	0.25	0.25	0^0

III. Numerical Results

Case 1: Coupled elliptic arc strips between grounded elliptic cylinders—In this case, twice the angle $(\pi/_2 - \eta_4)$ equal to α, say, represents the angle between the nearest edges of the coupled arc strips symmetrically located with respect to the line $\eta = \pi/_2$. It can be shown that $\eta_4 = \tan^{-1}(\coth \xi_o \tan \theta_4)$, where θ_4 is the angle made by the line joining the edge of the strip to the origin with the positive direction of the major axis of the ellipse. Using (9) and (10), the variation of even- and odd-mode impedances with α is computed for $\epsilon_1 = \epsilon_2 = 1.0$, $P_1/a_1 = 0.2$, $P_2/a_1 = 0.5$, $k/a_1 = 0.5$, $\eta_1 = 0.0$, and $\eta_4 - \eta_3 = 10^0$. The calculation is repeated for $\epsilon_1 = 9.9, \epsilon_2 = 2.56$. The results are presented in Fig. 2(a). The relation between $(\eta_4 - \eta_3)$ and θ_4 and θ_3, the angles made by the lines joining the two edges of the strip to the origin with the positive direction of this major axis of the ellipse can be obtained from the formula [4]

$$\eta_4 - \eta_3 = \tan^{-1}\left[\frac{\operatorname{Coth}\xi_o(\tan\theta_4 - \tan\theta_3)}{1+\operatorname{Coth}^2\xi_o\tan\theta_4\tan\theta_3}\right]. \tag{11}$$

If θ_o is the angle made by the radius vector passing through the center of the arc strip and $2\Delta\theta$ is the angular width of the arc strip, then

$$\theta_4 = \theta_o + \Delta\theta$$
$$\theta_3 = \theta_o - \Delta\theta.$$

Case 2: Coupled elliptic arc strips above a grounded elliptic cylinder—Assuming that $\xi_2 \to \infty$ and $\epsilon_2 = 1.0$, (9) and (10) for the even- and odd-mode impedances reduce to the form

$$Z_{oe} = 480\left\{\sum_{n=0}^{\infty}\frac{J_o^2\left(\frac{m_e\Delta}{2}\right)\operatorname{Sin}^2\left[m_e\left(\eta_3-\eta_1+\frac{\Delta}{2}\right)\right]}{(2n+1)[\epsilon_1\operatorname{Coth}[m_e(\xi_o-\xi_1)]+1]}\right.$$
$$\left.\cdot\sum_{n=0}^{\infty}\frac{J_o^2\left(\frac{m_e\Delta}{2}\right)\operatorname{Sin}^2\left[m_e\left(\eta_3-\eta_1+\frac{\Delta}{2}\right)\right]}{(2n+1)[\operatorname{Coth}[m_e(\xi_o-\xi_1)]+1]}\right\}^{1/2} \tag{12}$$

$$Z_{oo} = 240\left\{\sum_{n=0}^{\infty}\frac{J_o^2\left(\frac{m_o\Delta}{2}\right)\operatorname{Sin}^2\left[m_o\left(\eta_3-\eta_1+\frac{\Delta}{2}\right)\right]}{n[\epsilon_1\operatorname{Coth}[m_o(\xi_o-\xi_1)]+1]}\right.$$
$$\left.\cdot\sum_{n=0}^{\infty}\frac{J_o^2\left(\frac{m_o\Delta}{2}\right)\operatorname{Sin}^2\left[m_o\left(\eta_3-\eta_1+\frac{\Delta}{2}\right)\right]}{n[\operatorname{Coth}[m_o(\xi_o-\xi_1)]+1]}\right\}^{1/2} \tag{13}$$

Using (12) and (13), the variation of even- and odd-mode impedances with α of coupled elliptic microstripline is computed for $\epsilon_1 = 1.0$, $p_1/a_1 = 0.25$, $\eta_1 = 0^0$, $\eta_4 - \eta_3 = 10^0$, with $k/a_1 = 0.25$, 0.5, 0.75 as parameter. The calculations are repeated for $\epsilon_1 = 9.9$. The results are presented in Fig. 2(b).

Case 3: Circular arc strips between grounded circular cylinders—As $k/a_1 \to 0$, the ellipses and hyperbolas degenerate into circles and radial lines, respectively. It is found from 3(a), (b), and (c) that, for $k/a_1 = 0$

$$\xi_0 - \xi_1 = \ln\left[1+\frac{p_1}{a_1}\right] \tag{14a}$$

$$\xi_2 - \xi_o = \ln\left[\frac{a+p_2/a_1}{1+p_1/a_1}\right] \tag{14b}$$

and

$$\eta_3 = \theta_3 \tag{14c}$$

$$\eta_4 = \theta_4. \tag{14d}$$

Using (9) and (10) and (14a)–(14d), the variation of even- and odd-mode impedances with α is computed for $\epsilon_1 = \epsilon_2 = 1.0$, $p_1/a_1 = 0.2$, $p_2/a_1 = 0.5$, $\eta_1 = 0.0$, and $\eta_4 - \eta_3 = 10^0$. The calculation is repeated for $\epsilon_1 = 9.9$ and $\epsilon_2 = 2.56$. The results are presented in Fig. 3.

Case 4: Circular arc strips above a grounded circular cylinder—Using (12), (13), and (14a), the variation of even- and odd-mode impedances with α is computed for $\epsilon_1 = 1.0$, $\eta_1 = 0.0$, $\eta_4 - \eta_3 = 10^0$, $p_1/a_1 = 0.25$, 0.5, 0.75, and 1.30. The calculations are repeated for $\epsilon_1 = 9.9$. The results are presented in Fig. 4.

In all the above cases, calculation is also made for $\eta_1 = 10^0$ and the deviation from the results presented is negligibly small.

Case 5: Warped coupled strip and microstrip lines—The characteristic impedance of warped stripline of strip width W and the ground plane spacing b can be obtained from the results of case 3 by assuming that the radius of the cylindrical stripline is very large and the distance $b(=p_2)$ between the two ground planes remains finite. Then

$$\frac{\Delta}{2} = \beta_1 = \frac{\eta_4-\eta_3}{2} = \frac{W}{2a_1} = \frac{1}{2}\frac{W}{b}\cdot\frac{b}{a_1}.$$

Here, $b = p_2$

$$\frac{S}{2a_1} = \frac{\pi}{2} - (\eta_3 + 2\beta_1)$$

or

$$\eta_3 = \left(\frac{\pi}{2} - \frac{S}{2a_1} - \frac{W}{a_1}\right) = \left(\frac{\pi}{2} - \frac{1}{2}\frac{S}{b}\frac{b}{a_1} - \frac{W}{b}\cdot\frac{b}{a_1}\right)$$

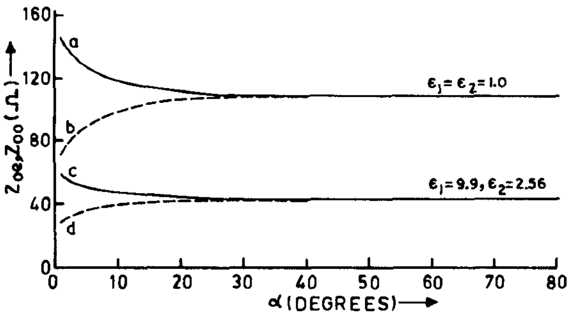

Fig. 3. Variation of even- and odd-mode impedances as a function of α for coupled cylindrical striplines, $\eta_4 - \eta_3 = 10^0$. —— even-mode characteristic impedance ---- odd-mode characteristic impedance.

Curve	ϵ_1	ϵ_2	p_1/a_1	p_2/a_1	K/a_1	η_1
a, b	1.0	1.0	0.20	0.50	0	0
c, d	9.9	2.56	0.20	0.50	0	0

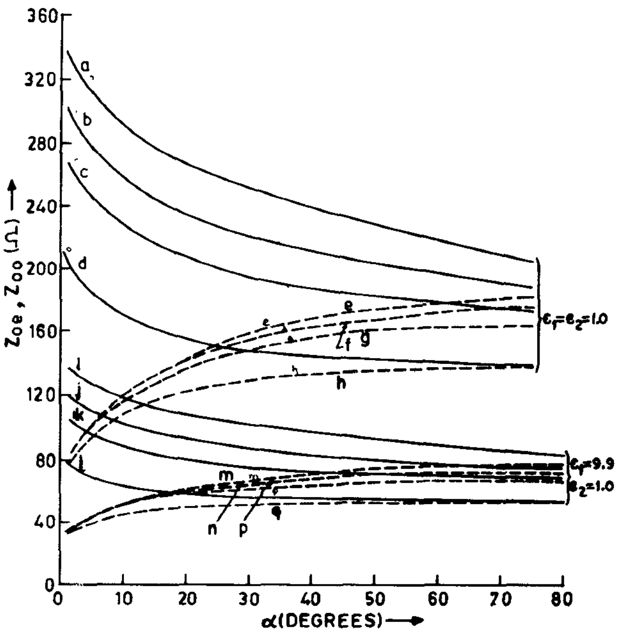

Fig. 4. Variation of even- and odd-mode impedances as a function of α for coupled cylindrical microstriplines, $\eta_4 - \eta_3 = 10^0$ —— even-mode characteristic impedance ---- odd-mode characteristic impedance.

Curve	ϵ_1	ϵ_2	p_1/a_1	K/a_1	η_1
a, e	1.0	1.0	1.30	0	0
b, f	1.0	1.0	0.75	0	0
c, g	1.0	1.0	0.50	0	0
d, h	1.0	1.0	0.25	0	0
i, m	9.9	1.0	1.30	0	0
j, n	9.9	1.0	0.75	0	0
k, p	9.9	1.0	0.50	0	0
$1, q$	9.9	1.0	0 25	0	0

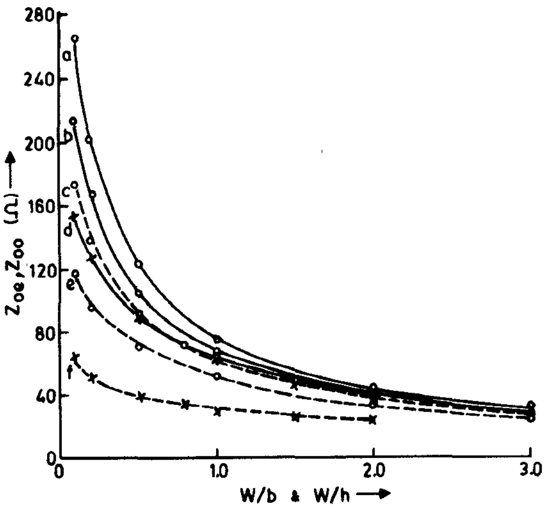

Fig. 5. Variation of even- and odd-mode impedances as a function of W/b and W/h for coupled warped striplines and microstrip lines, respectively.
∘ ∘ ∘ Cohn
× × × Bryand and Weiss
——— even-mode characteristic impedance
- - - - odd-mode characterisitc impedance

Curve	Description	ϵ_1	ϵ_2	S/b	S/h
a, e	Coupled warped striplines	1.0	1.0	0.1	—
b, c	Coupled warped striplines	1.0	1.0	0.5	—
d, f	Coupled warped microstriplines	10.0	1.0	—	0.2

where S is the edge to edge separation of the planar strips. When $\eta_1 = 0$

$$\eta_3 - \eta_1 + \frac{\eta_4 - \eta_3}{2} = \left(\frac{\pi}{2} - \frac{S}{2a_1} - \frac{W}{2a_1}\right).$$

It can be further approximated that

$$\xi_o - \xi_1 = \ln\left[1 + \frac{p_1}{a_1}\right] = \frac{p_1}{a_1} = A_1 \tag{15a}$$

$$\xi_2 - \xi_o = \ln\left[\frac{1 + p_2/a_1}{1 + p_1/a_1}\right] = \frac{p_2 - p_1}{a_1} = A_2. \tag{15b}$$

In this case, the expressions for the even- and odd-mode impedances of warped coupled striplines reduce to the from

$$Z_{oe} = 480 \cdot \left\{ \sum_{n=0}^{\infty} \frac{J_o^2(m_e\beta_1)\,\mathrm{Sin}^2\left[m_e\left(\frac{\pi}{2} - \beta_1 - \frac{S}{2a_1}\right)\right]}{(2n+1)[\epsilon_1 \,\mathrm{Coth}(m_e A_1) + \epsilon_2 \,\mathrm{Coth}(m_e A_2)]} \cdot \sum_{n=0}^{\infty} \frac{J_o^2(m_e\beta_1)\,\mathrm{Sin}^2\left[m_e\left(\frac{\pi}{2} - \beta_1 - \frac{S}{2a_1}\right)\right]}{(2n+1)[\mathrm{Coth}(m_e A_1) + \mathrm{Coth}(m_e A_2)]} \right\}^{1/2} \tag{16}$$

$$Z_{oo} = 240 \cdot \left\{ \sum_{n=0}^{\infty} \frac{J_o^2(m_o\beta_1)\,\mathrm{Sin}^2\left[m_o\left(\frac{\pi}{2} - \beta_1 - \frac{S}{2a_1}\right)\right]}{n[\epsilon_1 \,\mathrm{Coth}(m_o A_1) + \epsilon_2 \,\mathrm{Coth}(m_o A_2)]} \cdot \sum_{n=0}^{\infty} \frac{J_o^2(m_o\beta_1)\,\mathrm{Sin}^2\left[m_o\left(\frac{\pi}{2} - \beta_1 - \frac{S}{2a_1}\right)\right]}{n[\mathrm{Coth}(m_o A_1) + \mathrm{Coth}(m_o A_2)]} \right\}^{1/2}. \tag{17}$$

Using (16) and (17), the variation of even- and odd-mode impedances with W/b is computed for a symmetric stripline with $\epsilon_1 = \epsilon_2 = 1.0$, $p_1/a_1 = 0.005$, $p_2/a_1 = 0.01$, $\eta_1 = 0.0$, and $S/b = 0.1, 0.5$ as a parameter and the results are presented in Fig. 5. For the sake of comparison, the numerical results obtained by Cohn for a planar structure [7] are also presented in the same figure in the form of circles.

Following similar procedure, the variation of even- and odd-mode impedances of a microstripline is computed as a function of W/h $(h = p_1)$ for $\epsilon_1 = 10.0$, $\eta_1 = 0.0$, $p_1/a_1 = 0.01$, and $S/h = 0.2$ and the results are presented in Fig. 5. The results obtained by Bryant and Weiss [8] for a planar structure are also presented in the same figure in the form of crosses. There is a good agreement betweens the two sets of results.

IV. Conclusion

Agreement between the results obtained by the present method for the warped coupled strip- and microstriplines with those of Cohn for a planar symmetric coupled stripline, as well as Bryant and Weiss for a planar coupled microstripline, justifies the validity of the analysis. It is worthwhile to mention that computation is also made for $\eta_1 = 10^0$ and the deviation of corresponding numerical results from those for $\eta_1 = 0^0$ is negligibly small. The advantage of the method of analysis presented in this paper is that the same general formulation developed for coupled elliptical stripline structure can be applied to the coupled cylindrical strip- and microstriplines of elliptic and circular cross section. The method of analysis presented in the paper has enabled evaluation of even- and odd-mode impedance for different dielectrics on the two sides of the coupled strips.

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Computer-Optimized Multisection Transformers between Rectangular Waveguides of Adjacent Frequency Bands

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***Abstract*—Design data are given for multisection double-plane step transformers between *X*- (8.2–12.4 GHz), *Ku*- (12.4.–18 GHz), *K*-**

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